

Some Issues and Solutions for Complex Navigation Systems: Experience from the JRGPS Project

Vladislav Martínek
Dept. of Software Engineering
Charles University in Prague
Prague, Czech Republic
vladam@seznam.cz

Michal Žemlička
Dept. of Software Engineering
Charles University in Prague
Prague, Czech Republic
michal.zemlicka@mff.cuni.cz

Abstract—Navigation is a kind of application widespread especially in mobile devices. We can find complex navigation systems that combine two or more types of navigation. This can lead to a complex service, which can increase the efficiency of user’s movement. The user and even the path can be characterized by a large number of variables. This fact opens a problem of finding a path that will satisfy chosen variables.

We tried to point out interesting issues concerning combined navigation. For some of the issues we propose possible solutions. The solutions are based either on solutions from existing products or on our own experience from the JRGPS project supporting combination of public transport and walk.

Keywords-navigation; mobile device; GPS device; public transportation; complex navigation;

I. INTRODUCTION

Path searching applications (navigation systems) are very popular now: they are built in cars, they can be used to advise bikers, sometimes even for walkers. We can also find applications that can advise how to create complex service in public transportation. The main goal is to increase the overall effectiveness of users’ movement and to make the transportation more comfortable.

So there are applications dealing with the length of the path and speed limits, and there are also applications handling scheduled movement. There can be also other restrictions on the found path (reliability, safety, price, usability for people with limited movement abilities, or given movement speed). We suppose that good navigation system should be able to combine both the above mentioned approaches and at the same time take into account individual preferences and limitations of the user.

This paper is based on our experience in the development of an application for mobile devices that search the shortest path combining public transport and walking. We developed this application in a five-member team. We worked with timetables and map base for the city of Prague.

In the following text we will outline several problems and issues connected with developing complex navigation. We will also propose solutions used in our application. All the screenshots introduced in the article are from this

application. Finally, we will summarize advantages and disadvantages of combined navigation and propose intention of our future work.

II. STATE OF ART

In the area of pedestrian navigation used in urban location, there are at least two different approaches. First, it is a tourist guide, which is able to plan point-to-point path using walk exclusively. While navigating through the path, the navigation is able to highlight points of interest along the path. Second, it is a navigation that combines the walk with other types of transportation suitable for pedestrians. This application can still serve as a tourist guide, but the aim is often to serve as a path planning tool for everyday use.

On the other hand, the navigations can be differenced according to their dependency on the connection to the mobile operator. The off-line navigation system is able to work in areas where the connection is not available. For example, it can happen in subway train going between two neighbour stations. The path plan is computed by the mobile device itself in off-line navigation. The on-line navigation system has always actual data. The cost of data transfers from the operator can increase the cost of on-line navigation.

The complex navigation systems, that combine several types of transportation of pedestrians, are often realized as on-line services. For example, “Google Maps Navigation” for “Google Android” (<http://www.google.com/mobile/navigation/>) or “Navitime” (<http://www.navitime.com/>) are a kind of on-line services. On the other hand, “Nokia Maps” (<http://maps.nokia.com/>) provides an off-line navigation for pedestrians, but the complexity is so far limited for a combination of car and walk. Similar situation is with “TomTom” (<http://www.tomtom.com/>), “Navigon” (<http://www.navigon.com/>), “Mio” (<http://www.mio.com/>) and others, which are designed primarily for cars.

III. ISSUES

A. Navigation in a Scheduled Network

The services in transport network should follow a valid timetable. The separate parts of path in this network are

therefore predetermined in space and time. These parts can have a fixed starting moment or they can start periodically.

Path planning in scheduled network has one important property. The plan of the whole path can vary in space according to the starting time value. The consequence is that the path plan may differ significantly for two relatively close moments. See Figure 1.

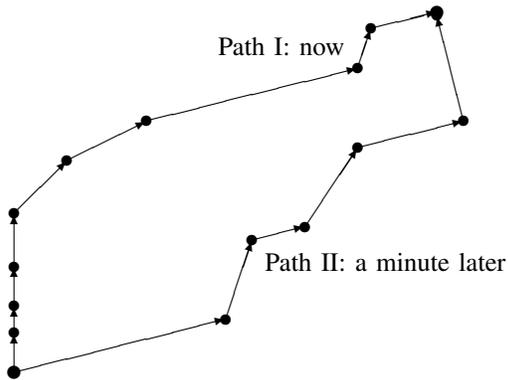


Figure 1. Path Plan Variability: The requirements for the highlighted path plans differ only by the starting time.

The validity of timetables is limited and should be kept up. In addition, there may be unplanned changes in the schedules. Temporary exceptions, technical problems, and other unforeseen events may affect the schedule as well.

B. Navigation of Walkers

It is not necessary to consider the current time when planning the walking route as a walker can typically start the journey at any time. But some sections of the walking path may be passable only at certain times of the day or in some days only. In this case, the current time should be considered. It is possible to find more complicated cases where the passage of some sections may be significantly more difficult and slower in certain periodical moments.

C. Combination of Different Navigations

1) *Fixed and Free Sections of Path:* When planning the combined path, it is necessary to distinguish sections fixed in time and free sections. The time spaces can appear between the parts of a planned path. These spaces can represent for example waiting for the service of public transport. The free parts of planned path could be moved in order to minimize the spaces or to satisfy the preferences of the user.

In order to properly combine the sections of path, it is necessary to know their length. In addition, the starting point for fixed sections is given and cannot be moved. The time is the key parameter for planning of the combined path. Individual public transport services are represented by fixed sections. Walking paths could be represented by both fixed and free sections. It is necessary to know the time needed to get through the section.

When planning the combined path, it is necessary to determine the length of each section before it is planned into the path. In the case of public transport, the duration is determined by the current time and the valid timetable of a service, which implements current section. The length of walk section is determined according to the time needed by the user to get through the section. In both cases the user defined parameters will be important for the planning. These parameters will affect the choice of sections and in the case of walk, they will also affect the length of individual sections.

Additional information about the character of a section is needed in the case of more complicated sections of walk path such as barriers or super elevated parts of path. Time needed to overcome such a section is based on current dispositions of a particular user.

2) *Combination of Different Search Networks:* Both public transport network and a network of pedestrian pathways may be very large and when combined, the total size of the searched network can grow over computation possibilities of mobile devices. Effectiveness of the overall planning is strongly dependent on the chosen solution.

Currently, portable devices have sufficient computing and memory capacity to handle the combination of path planning. It is necessary to choose an appropriate representation of data that will not exhaust the memory capacity of portable devices. This can lead to an application independent on the current availability of connection. Update of the timetables can be made when the connection or other mechanism for the update is available.

D. Path Reliability

Beside the path length, the reliability of path found might be the important parameter for a large number of users. To determine reliability of public transport services, we may require an additional data from the carrier. On the other hand, the reliability of the connection can be viewed as the frequency of services at the particular section.

Some fixed sections can be repeated in a relatively short periods of time. This behavior is similar to the behavior of free sections. For example, tube in the rush hour, when the arrivals are relatively frequent. If the user will miss such planned service, it does not have to be necessary to re-plan the whole path.

Interesting challenge might be planning a path, where the user may miss some of the services or even all of them. Respectively missing a service will lead to a minimal delay in following path sections.

When dealing with unreliable network, one of the approaches is to use approximative methods of finding optimal path. Two such methods are compared in [1].

1) *Response to Failures in the Network:* The failures of certain parts of network can occur in both public transport network and network of walk paths. Downtime can be

known in advance and then it should be included in the update. The failure may occur suddenly and then the user should have a possibility to change planning options to bypass actually unreachable part of network.

E. Appropriate Map Data

One of the major problems is the unavailability of appropriate map data for the planning of pedestrian paths. Most of the existing map data is not sufficiently detailed to pedestrians. First, the map base should include actual data for pedestrians – sidewalks, crosswalks, pedestrian zones, footpaths and other routes applicable to pedestrians. Spatial data should be in vector format, which can be easily used to create additional search structures.

1) *Crosswalks*: The map data should contain details of the crosswalks, or crosswalks with the signalling device where it is necessary to calculate the specific interval for the path section. The crosswalks are essential for legal crossing of the roads. In most countries, pedestrians are not allowed to cross the road in places close to the crosswalks. The situation is complicated when the sidewalk is bordered by a railing or other such barrier. To determine where the crossing of the road is acceptable outside the crosswalk is a separate problem. However, if the map data are not detailed enough, we cannot solve this problem anyway.

2) *Spatial Data*: The map data should contain information about elevation. Due to the variable dispositions of walkers the map data should contain information about various barriers. It is particularly important to distinguish the high thresholds of sidewalks or stairs, because for some user it is an obstacle, for some users it is not.

3) *Grade-separated Crossings*: Important information is the grade-separated crossings like bridges and subways. In the cases, where it is not possible to move freely between the levels of path, the incorrectly labelled crossings could lead to mistakes in navigation. The various levels of path need to be distinguished also when entering the starting point.

This information is relevant yet for the navigation in public transport itself. The path between refuges of one stop is often realized by a special crossing, like underpass or stairway. It is necessary to know the properties of the path connecting these refuges to correctly determine the length of transfer within the stop.

F. Combining Pedestrian and Public Transport Networks

Another problem is the combination of data for pedestrians and public transport network. Combination must be done in both directions.

1) *Street Refuges in the Map*: It is necessary to determine the nodes in the pedestrian network, from where the walkers can get in the public transport services. The ideal situation is when the map data contains the street refuges connected to the public traffic network. If the refuges are not in map data, it is necessary to add the node representing refuge

including the path connecting the refuge with the surrounding pedestrian network. The connection of the refuge into the pedestrian network is also important for the search of transfers between public traffic services.

2) *Mapping the Stops in Public Transport Network on the Street Refuges*: The network of public transport is typically created on the basis of routes of individual lines. Each stop in the itinerary of the line is identified by the name of the refuge, where the service stops. If there are several refuges of the same name, there is a problem of how to create a unique mapping between the names of refuges in the timetable and the refuges in the map. This problem does not occur if the street refuges from timetables are identified by geographical coordinates. The carrier should know the position of refuges, where its services stops.

G. Searching a Network of Public Transport

Paths search in the network of public transport is complicated by the fact that the value of each edge depends on the current time. Value of the edge is unknown until it is planned in some path.

1) *Unreliable Transfers*: If the transfer is realized within a single refuge, the following problem may occur. The timetable specifies only the expected time of departure from the refuge. In real traffic, there are deviations from the schedule. If two services are scheduled at the same time at the same station, it is not possible to guarantee which service arrives first in the real situation. If we consider the transfer between these services at the same minute, the transfer from the first service to the second is possible, but transfer in opposite direction cannot be guaranteed. The timetable does not determine the order of arrival of the services.

2) *Length of Platform*: Until now, we were considering stop refuge as a point. However, the length of platform can be noticeable. If the passenger has to walk across the long platform, it can lead into several minutes of delay against the planned path.

It is appropriate to walk across the entire platform only in the case, when the passenger is getting in the service on the opposite side than he is getting off. By walking across the platform into the appropriate position, the passenger can spare some time. This can lead into a faster transfer and the passenger can catch other following connection.

The problem makes sense only if the passenger arrives at the platform just in time of service departure. If the passenger comes earlier, then he is able to cross the platform into the appropriate position during his waiting for the service arrival. Likewise, if the passenger will be waiting for the following connection in the planned path, then the time needed to cross the platform after getting off the previous service could be subtracted from the waiting time.

IV. SOLUTION

A. Linking the network of walk paths and public transport network

In order to move freely between search network of the public transport and search network of walk paths, it is necessary to connect both networks in certain nodes. The connecting nodes should be the street refuges, where the passengers are getting in and off the services of public transport.

1) *Street Refuges in the Map:* The street refuges were missing in the map base available to us. We get the positions of street refuges from other source. It was therefore necessary to correct the coordinates of refuges and it was necessary to connect the refuges into existing network of walk paths.

2) *Mapping Stops in the Network of Public Transport to the Street Refuges:* In our case we did not have mapping of the street refuges of public transport to the places in map base. Nodes in the network of public transport are identified only by the stop names. For each stop, we had several refuges, representing different places in the map base. It was therefore necessary to distinguish the stops in the network of public transport, according to a service of public transport that is stopping at the current street refuge.

It was not possible to separate various street refuges of one stop in the public transport network. On the basis of practical experience we know that the lines of public transport stops at different refuges. To be able to perform the mapping, we had to manually record a set of services that passes the given street refuge. So we have assigned a line and direction to each refuge. However, this was not enough for unique mapping. More complex situations may appear if one line is going through more refuges of one stop. We had to add a lookout for one stop forward and one stop backward on the line route. Still more complicated situations may appear where this approach will not work. The mapping created this way requires maintenance in the case when the route of some line is changed. It is preferred that positions of the street refuges are identified directly in the data from the carrier.

B. Searching a Network of Walk Paths

On the basis of map data we had available, we created a search graph for the network of walk paths. The vertices of this graph are crossing or closing of some polyline. Specific nodes of this graph are the street refuges, which hold the identification of corresponding stop in the public transport network. So it is possible to move continuously from the walk paths search network to the search network of public transport.

Each polyline is represented by two oriented edges in opposite directions to each other. The value of the edges determines the duration of walking, which usually depends



Figure 2. Starting position selection

The cross marks the selected starting position. The nearby walk paths are highlighted. The chosen section of walk path is the one closest to the starting position.

on the walking speed and segment length. The problem occurs in sections with superelevation or some kind of barrier and at the crossings. The edge value may vary depending on the direction and can be even dynamic. In these cases, the details from map base are very important, because they determine the duration of walking in the given section. The duration should be parameterized by the actual dispositions of the user.

1) *Network Reduction:* When converting vector data to a network, it is possible to make a simple reduction of the vertices, where there is no branching of the graph.

2) *Entering Position on the Map:* When entering the starting and target position on the map it is necessary to determine precisely the walk path that is closest to the user. We do not know the path from general position to the closest walk path, so we approximate it by a direct line. The selection of the closest walk path is given up to the user (see Figure 2). This choice can be complicated and if the automatic approximation fails, the user can make a correction immediately according to his knowledge of the current location. In the worst case, the user will rely on the automatic choice.

C. Searching a Network of Public Transport

We have created the search graph for the network of public transport from available timetables. Vertices represents stop refuges, each refuge has its position on the map, from where the user can continue in walk path.

Each edge represents a possibility to take a service to the next stop on the route of the line. The edges are characterized by a value, which determines the duration of travelling to the neighbouring refuge. But if the user should get on a service in the planned path, then it is necessary to add a waiting time to the value of currently planned path. This time is derived from the current time (when the user gets to

the stop according to the path plan) and valid timetable of the service he is waiting for.

Other approach could be representing every departure of a service as a vertex. This approach is robust for complex scenarios but not necessary in our case. Moreover this approach shows significantly lower performance as described in [2].

1) *Graph Reduction*: An interesting method of rail network reduction is described in [3]. Under certain conditions it can be used to reduce the network of city public transport and so speed up the planning significantly. Such reduction is generally an NP-hard problem. Fortunately, it is possible to solve the task in acceptable time [4]. Using approach based on this reduction the computational complexity will decrease to the level, where the path planning itself can be computed on portable devices in reasonable time [5].

The timetables determines the dynamic part of the graph. The representation of timetables should deal with the irregularities of a real world. One of the interesting approaches is described in [6].

2) *Unreliable Transfers*: If in the planned path is a transfer inside one refuge between two lines, which are leaving in the same minute, we are not able to ensure the order of services in most cases. We handle the situation by searching the departure time of the following service from the next minute from actual time. So it cannot happen, that we plan a transfer, which the user cannot make.

D. Path Planning in Combined Network

If we consider the task in general case, we are given the starting and the target position on the map and we want to find the path to connect them. When planning, we will first plan walking routes from the starting position to all public transport stops in a particular area. Similarly, we plan walking routes for the target position. Walk paths should be searched in the opposite direction, if for example super elevation is needed to take into account.

This way the general task is reduced to the task of planning connection in the public transport network. When planning transfer between services, it is still needed to use the walking graph to determine the transfer duration.

1) *Searching Walk Path to the Stop*: When searching for walking path from the starting position, all stop refuges in given area are relevant to us. The passenger can use public transport service after reaching the refuge. The user can adjust the size of area, according to how far he is willing to walk. Due to this limitation it is not necessary to search the entire network of walk paths, but only a relatively small part.

Due to the breadth first search, we are able to find all paths from the starting position to all street refuges in the area at once. The following search is made in the network of public transport, where the starting positions are the street

refuges reached by walk, and their initial estimation is the length of the walk path from the starting point.

2) *Precomputation of Transfers*: To avoid searching over both networks simultaneously, we performed precomputation of walk transfers and added special edges representing transfers into the network of public transport. As a result during the search we do not have to leave the search graph for public transportation, so the overall branching of computation is decreased.

As it is a precomputation, it is necessary to specify the maximum length of walk transfer between services while creating search graphs. If we do not limit the length of walk transfer, it would lead to unbearable increment of branching of public transport search graph. It is unreliable for both computational and memory demands. On the other hand, if we choose too strict limit of walk transfers, the path planning possibilities would be reduced. Some of the transfers would not get into the search graph due to precomputation. The limit of walk transfer is determined during the compilation of data, mostly based on experience and tests.

3) *Searching Connection in Public Transport*: Due to previous steps, the planning of path is reduced into the searching path in the enriched graph of public transport. A specific part of this search is that in addition to previously found paths it is necessary to remember the current time. The waiting time for a service changing is determined according to the current time and timetable that is currently valid. On the basis of the current time other parameters of the dynamic network could be determined.

The user parameters and preferences should be taken into account when planning the path. In particular, the maximum number of transfers, the maximum length of walk section (the user is willing to walk continuously only for a certain distance), walk speed and more.

E. User Preferences

Users may have very different movement dispositions. This can significantly affect planning of the path. We tried to take into account at least basic user parameters.

1) *Walk Speed*: All walk edges in search graphs are valued by the duration rather than length. Except special edges, this value is a walking time. This is determined by the length of walk section, which is represented by the edge, and the default walking speed 5km/h. The appropriate correction of length of the walk section is performed only if the user changes the default walking speed.

Special edges are distinguished by additional indication of the character of the section, which is represented by the given edge. The value of special edges is not affected by walking speed.

2) *The Maximum Length of Continuous Walking Section*: This parameter is added especially for precomputation of walking transfers, which are theoretically limiting possibilities of walk transfer. Practical importance to the user is if he

set the limit to a value less than or equal to the limit used in precomputation. At the same time, this parameter limits the size of the area for searching walk paths to the closest street refuges.

F. User Places

In everyday use of our navigation some set of places will be used frequently as a starting or target points of path search. These places can represent home, school, work, etc. In these cases, most users already know the walking path, and knows the time that it takes to the stops in certain area. Therefore, we have introduced a possibility to predefine these places, including duration of walk paths to the stops. Predefined positions reduce the time needed for user input and increase user comfort of the application.

G. Reliability of the Path

A sudden reduction of the transport network may occur during the travel, for example due to a technical fault of the route or vehicle. In this case, the current path plan may be irrelevant and needs to be recomputed according to the new situation.

1) *Excluding Line:* In our application we allow the user to react to a situation similar to the exclusion of a particular line, which is affected by the failure. Any number of lines can be excluded from the search. After that, the path will be planned using other routes.

2) *Excluding Section:* Failure of a part of the network can affect both public transport and walk paths network. In both cases it is necessary to allow the user to identify the affected part of the network and reschedule path plans another way. This action may require experienced user and we do not solve it in our application.

V. USE CASES

A. Path Plan Dependency on Starting Time

The following example of path plan shows two paths between the same places. The only difference on the input of planning is the starting time. The first path starts only one single minute earlier. This situation shows, how critical for the resulting path plan is the current time and reliability of schedule of public transport services. See Figure 1.

In the case of failure in the network, the situation will be similar. The new recomputed path will have a significantly different plan in comparison with the original path plan leading through the unavailable part of network.

B. Example of Advantageous Walk Transfer

Benefits of combination walk paths with the public transport will appear in situations where it is better to walk to a distant stop concerning the overall length of path. The Figure 3 shows initial walk segment of path plan leading from the starting position to a remote stop.

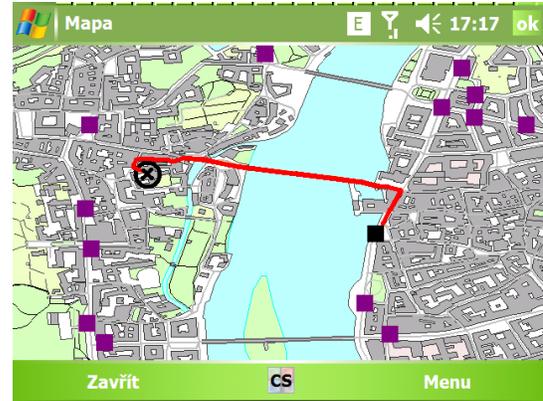


Figure 3. Advantageous Walk Transfer

For overall path length, it is advantageous to consider the possibilities of travelling from all stops in certain area.

Sometimes the long initial walk section could lead to a shorter path, especially in town areas separated by river or other obstacles.

VI. FUTURE PLANS

A. Multicriteria Path Search

So far we have discussed only finding the time shortest path with some restrictive conditions. Requirements on the final path plan may be various and may not always be strict conditions. To be able to take into account various preferences will need to perform multicriteria search on a combination of networks. One of the promising approach is to find Pareto-optimal solution for multicriteria path search, which is studied for railway networks in [7].

Computational complexity may exceed the possibilities of portable devices. It is therefore appropriate in the context of this approach to consider a different approach to the overall solution.

1) *Reliability of the Path Found:* One of the characteristic features of planning in public transport is the fact that the timetables are only a prescription for service scheduling. In real cases the services can be delayed or cancelled. In the case some of this situation is frequent, we can count on a certain probability that the service comes on time or will have a certain delay. If we know these probabilities, we can take into account the reliability of the connection when planning the path. Alternatively, if the user requires a reliable path, we can adjust the planning to handle the most probable delays.

Moreover, it can cause the following situation. A service with less frequent intervals can occur in the path. Missing this service would mean a serious time loss for the user. In this case, it is appropriate to plan the route so that even in bad traffic conditions with high probable delays, it would be possible to guarantee a high probability of catching the critical service.

2) *Points of Interest*: Like in the case of ordinary tourist navigation, we should also be able to add points of interest. So the route plan could be adapted to the requirement to visit a point of interest or a category of points of interest, which is located closest to the direction of the planned path. These ideas are based on the assumption of the use of multicriteria path planning.

VII. CONCLUSION

A. Advantages of Combination of Two Different Types of Navigation

1) *Path Efficiency*: Combining the two networks gives us much more scheduling options than if we use only one type of navigation. Moreover, for walk sections we have far more information than if we used only the navigation in public transport. The resulting path plan does not have to estimate the transfers' duration, but is more accurate, because the transfer path is known. This allows us to plan more efficient and more reliable path.

2) *Environmental Aspect*: We are trying to offer comfortable and accurate planning in the city using public transport to a wide range of users. This way we are increasing the comfort of the use of public transport and the level of transport-related services. The more users will use public transport at the expense of less ecological alternatives, the smaller will be the impact of urban transport on the environment.

B. Disadvantages of Combination of Two Different Types of Navigation

1) *Different Planning*: We try to combine two very different networks. In each of these are different rules and heuristics, which can be successfully applied in one network, but may not be valid in the other. It is therefore necessary to separate the search. On the other hand, the total travel plan should meet the common criteria. To achieve this, it is often needed to use a different mechanism in each network.

2) *Different Sources of Data*: For a network of public transport we need data of timetables and data of the positions of stops refuges. Walk network needs map data, including details needed for navigation of walkers. The application needs data from two different entities. In the case of commercial deployment of applications, the question "how to split the profit?" arises.

C. Available Data

While developing our application, we had available data for the city of Prague. Map data provided to us "the Czech Office for Surveying and Mapping". Although the map data were not initially designed for the operation of navigation, we managed to adapt mechanisms working with them so that our application was able to bring reasonable results.

The available data have shown that the operation of the application is not limited by memory or computing capabilities of portable devices. In addition a limited connectivity is sufficient to keep the data updated. For most European cities the search parameters should be comparable, excluding much larger cities like Paris, London, or Moscow.

D. Real Life Consequences

Using the JRGPS application we have learned that it can be reasonable to change slightly our habits: In some cases it is better to go on foot instead of waiting for public transport and in some other cases it can be advantageous to change entry or leaving stop.

ACKNOWLEDGMENT

This paper was partially supported by the Czech Science Foundation by the grant number 201/09/0983.

REFERENCES

- [1] J. Koszelew, "Two methods of quasi-optimal routes generation in public transportation network," in *CISIM '08: Proceedings of the 2008 7th Computer Information Systems and Industrial Management Applications*. Washington, DC, USA: IEEE Computer Society, 2008, pp. 231–236.
- [2] E. Pyrga, F. Schulz, D. Wagner, and C. Zaroliagis, "Efficient models for timetable information in public transportation systems," *J. Exp. Algorithmics*, vol. 12, pp. 1–39, 2008.
- [3] K. Weihe, "Covering trains by stations or the power of data reduction," in *Proceedings of "Algorithms and Experiments" (ALEX98)*, R. Battiti and A. A. Bertossi, Eds., 1998, pp. 1–8.
- [4] A. Liebers and K. Weihe, "Recognizing bundles in time table graphs - a structural approach," in *Algorithm Engineering*, ser. Lecture Notes in Computer Science, S. Näher and D. Wagner, Eds., vol. 1982. Springer, 2000, pp. 87–98.
- [5] V. Martínek and M. Žemlička, "Speeding up shortest path search in public transport networks," in *DATESO 2009*, K. Richta, J. Pokorný, and V. Snásel, Eds. Prague, Czech Republic: Czech Technical University in Prague, 2009, pp. 1–12.
- [6] R. Kasperovics, M. H. Böhlen, and J. Gamper, "Representing public transport schedules as repeating trips," in *TIME '08: Proceedings of the 2008 15th International Symposium on Temporal Representation and Reasoning*. Washington, DC, USA: IEEE Computer Society, 2008, pp. 54–58.
- [7] M. Müller-Hannemann and K. Weihe, "On the cardinality of the pareto set in bicriteria shortest path problems." *Annals OR*, vol. 147, no. 1, pp. 269–286, 2006.